METHOD FOR REGISTERING THE ACTUAL DESCRIPTION OF A MEASURED OBJECT WITH A NOMINAL DESCRIPTION

BACKGROUND OF THE INVENTION

[0001] The present invention relates to a method for registering the actual description MI of an object under measurement with a nominal description MS of the object under measurement in which actual description MI describes the actual geometric dimensions, the actual position and the actual orientation of the object under measurement and in which defined geometric dimensions, a defined position and a defined orientation are specified via nominal description MS for the object under measurement. The elements of actual description MI are measured and the elements of nominal description MS are predefined.

[0002] In general, a registration is used to link data which is present in different coordinate system. At this point, the setpoint/actual value comparison between CAD data and measured data should be mentioned as an example of such a linkage. The CAD data describes a design element and, consequently, represent the nominal description of a workpiece which has been manufactured on the basis of the aforementioned CAD data. The measured data represents the actual description of the manufactured workpiece which constitutes the object under measurement. The CAD data exists in the so-called "design coordinate system" whereas the measured data exists in the measuring or workpiece coordinate system. The intention is for a transformation describing the interrelationship of the involved coordinate systems to be determined with the aid of the registration.

[0003] In the following, two examples of application for a registration will be described with reference to Figures 1 through 4 as well as 5 and 6.

[0004] Figures 1 through 4 depict the preparation of a setpoint/actual value comparison by registration using subsequent transformation. Fig. 1 shows the CAD description of a turbine blade 1 featuring a base 2. Defined in the CAD description are alignment points 3 which predefine the positions which the transformation is intended to approximate the best way possible. Alignment points 3 can be allocated a weighting factor which increases or else

reduces the weight of an alignment point 3. Thus, in the depicted example, at one time base 2 and, at another time, blade region 4 of turbine blade 1 can be given more weight and thus be approximated to a greater degree by the registration. In the present case, base 2, as the mounting location, is taken to be highly binding. The measured data is depicted in Fig. 2. Fig. 3 shows the measured data together with the CAD description so that the existing misorientation of the object under measurement with respect to the CAD description is clearly discernible. A direct comparison between the two data sets, namely of the nominal description in the form of the CAD description and the actual description in the form of the measured data is still not easily possible in this manner. With the aid of registration, it is now possible to find a transformation which enables such a comparison. Fig. 4 shows the situation after a suitable transformation has been applied to the measured data. This transformation yields the desired excellent correspondence in base region 2 of turbine blade 1. The here existing error of form of the object under measurement in blade region 4 can now be determined.

[0005] Figures 5 and 6 depict a sensor calibration and a sensor alignment by registration, respectively. Fig. 5 shows the measurement set-up for the calibration of a contour sensor 5. In lieu of a workpiece, a test means 6 is measured whose CAD description exists in the measuring coordinate system of sensor 5. The measured values of calibrated sensor 5 are intended to lie within a tolerance field on the intersection between test means 5 and the measuring plane of sensor 5 which is shown with a dotted line. If this is not the case, which is shown here with a solid line, an undefined deviation between the measured test means 6 and the CAD description of test means 6 is detected. However, since the configuration of test means 6 is known, the deviation that has arisen is attributed to the fact that sensor 5 is not calibrated any more. The relationship of the coordinate systems of test means 6 and sensor 5 is no longer known sufficiently accurately.

[0006] Fig. 6 shows how, via a registration of the measuring points onto the CAD description of test means 6, a transformation is determined which minimizes the calibration error. It is not absolutely necessary for the contour of test means 6 to be exactly fitted in because a contour

error, that is a form deviation of test means 6 from its CAD description is of no importance here. In the present case, it is possible to determine a transformation which is exclusively composed of a translation and a rotation in the sensor plane. At this point, it should be mentioned that the sought transformation is generally not limited as in the example which is described here. Via the transformation, the misorientation of the sensor is determined as well. To calibrate the sensor, it is either possible to compensate for the misorientation by calculation or to realign the sensor in accordance with the ascertained transformation. The deviations between measured test means 6 and the CAD description of test means 6 represent a measure for the quality of the sensor.

[0007] In practice, for example, a so-called "coordinate measuring instrument measurement" is carried out for aligning the object under measurement. In the process, the measuring points on the basis of which the object under measurement is to be aligned are predefined by a test plan. These measuring points are derived from the appertaining reference coordinates of the CAD data. The alignment of the object under measurement is generally carried out according to the 321 method which will be described in the following:

[0008] - A spatial direction is determined as the first axis of the workpiece coordinate system to be generated. Depending on the type of the workpiece, this can be the orientation of a plane which has been defined by at least three measuring points or else a cylinder axis which has been defined by n measuring points.

[0009] - The second axis of the workpiece coordinate system is determined by at least two measuring points which can lie in a plane perpendicular to the first axis of the workpiece coordinate system. If necessary, the projection of the measuring points onto this plane can also be used.

[0010] - Thus, the third axis of the workpiece coordinate system is determined as well because it is oriented perpendicularly to the two already determined axes.

[0011] - Finally, the zero point of the workpiece coordinate system is defined as well, for example, also with the aid of a measuring point.

[0012] - If the workpiece coordinate system and the design coordinate system are not

identical, it can be required for the workpiece coordinate system to be transformed in an additional step to obtain the design coordinate system. To this end, it is possible, for example, to shift the zero point of the workpiece coordinate system in a predefined fixed translation.

[0013] In practice, the above-described method turns out to be relatively inflexible since, here, it is only possible to evaluate measured data which has been acquired at the measuring points specified by the test plan. Here, an alignment of the object under measurement on the basis of an overdetermined set of measured data is not possible. Besides, coordinate measuring instruments can be used in manufacturing processes only to a limited extend because of the execution time required for the alignment and the actual measurement. Thus, when working with cycle times of 5 minutes and less, 100% control can no longer be attained.

SUMMARY OF THE INVENTION

[0014] An object of the present invention is to develop and refine a method for registering of the type mentioned at the outset in such a manner that measured data representing an overdetermined image of the object under measurement can be processed as well, and that the execution time required for determining the alignment of the object under measurement allows the method according to the present invention to be used on-line in a manufacturing process.

[0015] The present invention provides a method for registering the actual description MI of an object under measurement with a nominal description MS of the object under measurement. The actual description MI describes the actual geometric dimensions, the actual position and the actual orientation of the object under measurement, and defined geometric dimensions, a defined position and a defined orientation are specified via the a nominal description MS for the object under measurement in which the elements of the actual description MI are measured and the elements of nominal description MS are predefined. A transformation T aligning the object under measurement according to its nominal description

is determined in an iterative method.

[0016] The transformation is determined using the iterative method:

(1) by determining an initial transformation T_{init} which is used for mapping the actual description MI, resulting in a transformed actual description MIt, and by mapping the nominal description MS with the inverse T_{init}^{-} of initial transformation T_{init}, resulting in a transformed nominal description MSt,

(2) by determining target elements from nominal description MS for selected elements of transformed actual description MIt, and by determining target elements from actual description MI for selected elements of transformed nominal description MSt,

(3) in that the target elements of MIt are transformed with the inverse T_{init}^{-} and form the target elements for the selected elements of actual description MI; and in that the target elements of MSt are transformed with the initial transformation T_{init} and form the target elements for the selected elements of nominal description MS,

(4) in that, for determining the quality of transformation T_{init}, both a scalar clamping error is determined from the determined selected elements of transformed actual description MIt and corresponding target elements of nominal description MS, and a scalar clamping error is determined from the determined selected elements of actual description MI and corresponding target elements of transformed nominal description MSt,

(5) by modifying transformation T_{init} as a function of its quality in an exploratory method, and by proceeding with the modified transformation according to steps (1) through (5) until a termination criterion for the exploration is reached.

[0016] The method according to the present invention is considerably less sensitive with respect to measured-value acquisition than the coordinate measuring instrument measurements known from practice for which the measured-value acquisition must take place at measuring points which are predefined by a test plan. In contrast, the method according to the present invention permits the processing of measured data which represents an overdetermined image of the object under measurement. According to the present invention,

in fact, the elements of the actual description which best correspond to the elements of the nominal description are determined from this measured data, which will be described in detail within the scope of a detailed explanation of the method according to the present invention. Thus, measured values from 1D, 2D and 3D sensors can be used jointly within the scope of the method according to the present invention. In this context, it is possible to use sensors having different functional principles as, for example, sensors working in a tactile, inductive, or optical manner. The method according to the present invention can be flexibly and individually configured by suitable selections or inputs according to the specific purpose of use, which is additionally promoted by a modular design of the method according to the present invention. Moreover, the modules of the method according to the present invention can be used individually or also in combination for solving further metrological problems such as in quality assurance, sensor testing or sensor calibration.

[0017] As a general principle, the teaching of the present invention can be implemented and advantageously refined in different way as described for example in the dependent claims.

BRIEF DESCRIPTION OF THE DRAWINGS

[0018] The following detailed explanation of a preferred method according to the present invention is explained with reference to the following drawings, in which:

[0019] Figures 1 through 4 show the preparation of a setpoint/actual value comparison by registration with subsequent transformation (as discussed above).

[0020] Figures 5 and 6 depict a sensor calibration and a sensor alignment by registration, respectively, (as discussed above).

[0021] Figures 7 through 11 show a method for determining a clamping error as a measure for the deviation of the actual description of an object under measurement from its nominal description.

[0022] Figures 12 through 14 show a method for determining corresponding elements, in particular spatial points, in two descriptions of an object under measurement.

[0023] Figures 15 through 20 show a method for determining a transformation for aligning an object under measurement according to its nominal description.

DETAILED DESCRIPTION

[0024] Initially, for easier understanding of the method according to the present invention, the determination of a scalar clamping error as it is determined in step (4) of the method according to the present invention will be explained in the following with reference to Figures 7 through 11. The clamping error is a measure for the deviation of the actual description of an object under measurement from its nominal description, that is a measure for the misorientation of an object under measurement with respect to its nominal description.

[0025] The clamping error is determined on the basis of:

- a nominal description MS of the object under measurement, that is a shape description of the object under measurement in its correct clamping position, the nominal description existing in the form of points in space, it being possible for each point to be allocated a direction in space,
- an actual description MI of the object under measurement, that is the description of the actual clamping position of the object under measurement, the actual description existing in the form of points in space, it being possible for each point to be allocated a direction in space,
- a clear correlation between the points of the nominal description and the points of the actual description,
- a rule for conditioning the measured data (constraint error measure determination), namely for determining the error measure for each pair of corresponding points of the nominal description and actual description,
- a rule for bringing together the error measures determined for each pair of corresponding points of the nominal description and actual description (constraint

clamping error determination).

[0025] Actual description MI includes features which are obtained from the totality of measured data. These features can be determined automatically or interactively by the user (selected values). They can also be automatically generated as target values for MS during the fitting-in, which will still be explained in greater detail in the following. The elements of actual description MI can describe, for example, single measuring points, measuring points averaged by modeling, center points of circles and spheres, circle or cylinder axes, the orientation of a plane, etc. In this context, geometric elements such as plane, circle, etc. are fitted into the set of measuring points in that they constitute a description of the measuring points which is reduced in data and generally afflicted with less noise as well. Besides, it is possible to form further geometric elements for the measured data without these geometric elements being elements of actual description MI. Geometric elements can also be derived from measured data on measuring aids. Measuring aids are, for example, pins inserted into bore holes, balls inserted into punched openings, or also clamping devices, etc.

[0026] Analogously, nominal description MS can include features such as single points on CAD description, axes of cylinders etc., center points of circles or spheres etc. with or without indication of direction. On the basis of the existing design, new or altered elements can be generated and used here as "aids". Nominal description MS contains selected values and target values for MI. The target values extend in each case the other set, that is the target values for MI extend nominal description MS such as the target values for MS extend actual description MI.

[0027] Advantageously selected as spatial points P_{si} of nominal description MS are points which each lie in one of the defined surfaces of the object under measurement. These spatial points P_{si} can then be allocated in each case the normal to the corresponding defined surface of the object under measurement in spatial point P_{si} as spatial directions R_{si}. Correspondingly, it is advantageous if points in one of the surfaces of the object under measurement are determined as spatial points P_{li} of actual description MI. These spatial points P_{li} are then

allocated in each case the normal to the corresponding defined surface of the object under measurement in spatial point P_{li} as spatial directions R_{li}.

[0028] At this point, it should be mentioned that spatial points P_{si} of nominal description MS and spatial points P_{li} of the actual description can be allocated feature elements alternatively or also in addition to spatial directions. Here, possible geometrical feature elements include a circle, a cylinder, or a sphere, which represent the configuration of the object under measurement in specific regions.

[0029] Fig. 7 depicts nominal description MS of an object under measurement with corresponding actual description MI of the object under measurement. Here, nominal description MS is composed of spatial points P_{si} and spatial directions R_{si}. The actual description is composed of spatial points P_{li} and spatial directions R_{li}.

[0030] Scalar clamping error F of this overall configuration is led back to the determination of an error measure F_i for each pair of corresponding points from nominal description MS and actual description MI. Figures 8 through 11 show the different ways of determining the error measure, depending on the availability of directional information on spatial points P_{si} and P_{li}.

[0031] In Fig. 8, spatial point P_s of nominal description MS is allocated spatial direction R_s while no spatial direction is allocated to corresponding spatial point P_l of actual description MI. Vectorial difference A in the X, Y and Z directions is now determined as shown. Via directional information R_s, it is also possible to break down vectorial difference A into a vector component AN perpendicular to the plane given by spatial point P_s and spatial direction R_s and into a vector component AL within this plane. For the constellation shown in Fig 8, it is thus possible for vectorial difference A to be described in vector components AX, AY, and AZ.

[0032] In the case of the situation shown in Fig. 9, spatial point P_l of actual description MI is allocated spatial direction R_l while no spatial direction is allocated to corresponding spatial

point P_s of nominal description MS. Here, vectorial difference A can analogously be described in vector components AX, AY, and AZ or in vector components AN and AL, AN representing the vector component of vectorial difference A perpendicular to the plane given by spatial point P_l and spatial direction R_l, and AL representing the vector component of vectorial difference A within this plane.

[0033] If no spatial direction is available for any of spatial points P_s and P_l, as is shown in Fig. 10, then vectorial difference A is represented in its vector components AX, AY, and AZ with a given coordinate system K. Via a combination of AX, AY, or AZ, it is thus possible to specify the difference in one of the main planes of coordinate system KXY, KYZ, and KXZ, or the difference along one of axes KX, KY, or KZ of the coordinate system.

[0034] If both of spatial points P_s and P_l are allocated a spacial direction R_s and R_l, then vectorial difference A can be determined either as described in connection with Fig. 8 or as described in connection with Fig. 9. In this case, moreover, an angle error w can be determined as shown in Fig. 11.

[0035] A scalar error measure for the mutually corresponding spatial points P_s and P_l is determined from vector components AX, AY, AZ, AL and/or AN and from angle error w. This error measure includes a distance component and, possibly, an angle component. Both the distance component and the angle component can be weighted with a weighting factor Ga or Gw, respectively, prior to adding up these two components. Selected as distance component is a vector component AX, AY, AZ, AN, or AL, or a combination of several of these vector components. Resulting therefrom by vectorial addition is a differential vector D which is representative of the distance error and whose length is determined as distance component of the scalar error measure. If a spatial direction was available during the determination of vector components AX, AY, AZ, AL and AN, then the distance component of the scalar error measure can also be allocated a plus or minus sign. The sign is determined in accordance with Figures 8 or 9. The spatial point, which is allocated a spatial direction, defines a plane in conjunction with this spatial direction. If the spatial point corresponding to

this spatial point lies above or within this plane, then the distance component is allocated a positive sign +1. In the other case, the sign of the distance component is negative, -1. Angle error w (See Fig. 11), as a scalar value, constitutes the angle component of the scalar error measure without further transformation.

[0036] The error measure determination for a pair of corresponding spatial points of the nominal description and the actual description of an object under measurement can thus be summarized as follows:

[0037] Given are:

1. a spatial point P_{si} of the nominal description and, possibly, a spatial direction R_{si} allocated to this spatial point P_{si} (spatial direction R_{si} can also be absent),
2. a spatial point P_{li} of the actual description corresponding to spatial point P_{si} of the nominal description and, possibly, a spatial direction R_{li} allocated to this spatial point P_{li} (spatial direction R_{li} can also be absent),
3. information on which of vector components AX_i, AY_i, AZ_i, AN_i, AL_i is intended to form differential vector D_i,
4. weighting factors Ga_i for the distance error and Gw_i for the angle error,
5. tolerance fields for the distance error and the angle error.

[0038] The required error measure for the pair P_{si}, P_{li} is then determined as

$$F_i = Ga_i \bullet \text{length}(D_i) + Gw_i \bullet w_i$$

[0039] If it is not possible for the initial data specified under 1. and 2. to be processed with the calculation rules specified under 3. and 4., then an error measure

$$F_i = 0$$

is specified.

[0040] If length(D_i) lies outside of the tolerance field for the distance mentioned under point

5., then the distance component is ignored and the error measure is determined as

$$F_i = Gw_i \bullet w_i$$

[0041] If w_i lies outside of the tolerance field for the angle mentioned under point 5., then the angle component is ignored and the error measure is determined as

$$F_i = Ga_i \bullet \mathrm{length}(D_i)$$

[0042] If both length(D_i) and w_i lie outside of the tolerance fields mentioned under point 5., then the error measure is determined as

$$F_i = 0$$

[0043] As already mentioned, scalar clamping error F of the overall configuration, that is between the nominal description and the actual description of an object under measurement altogether, is led back to the determination of an error measure F_i for each pair of corresponding points from nominal description MS and actual description MI.. To determine scalar clamping error F, all error measures F_i are brought together. There are, inter alia, the following ways to do this:

[0044] • The mean square deviation from scalar clamping errors F_i related to the expected value zero, the smallest possible error measure for a pair of corresponding spatial points

$$F = \mathrm{SQRT}(\mathrm{SUM}(1, n, Ga_i \bullet \mathrm{length}(D_i) \bullet \mathrm{length}(D_i) + Gw_i \bullet w_i \bullet w_i))) / \mathrm{SUM}(1,n,(Ga_i + Gw_i))$$

where n is the number of pairs for which a clamping error F_i could be determined.

[0045] • The mean deviation from the absolute values of scalar clamping errors F_i

$$F = \text{SUM}(1, n, (\text{ABSOLUTE VALUE}(Ga_i \bullet \text{length}(D_i)) + \text{ABSOLUTE VALUE}(Gw_i \bullet w_i))) \,/\, \text{SUM}(1, n, (Ga_i + Gw_i))$$

where n is the number of pairs for which a clamping error F_i could be determined.

- The maximum deviation from the absolute values of scalar clamping errors F_i

$$F = \text{MAXIMUM}(\text{ABSOLUTE VALUE}(F_1), (\text{ABSOLUTE VALUE}(F_2), \ldots, (\text{ABSOLUTE VALUE}(F_n))$$

- The maximum positive deviation of scalar clamping errors F_i

$$F = \text{MAXIMUM}(F_1, F_2, \ldots, F_n)$$

- The maximum negative deviation of scalar clamping errors F_i

$$F = \text{MINIMUM}(F_1, F_2, \ldots, F_n),$$

where n is in each case the number of pairs for which a clamping error F_i could be determined.

[0046] On the basis of Figures 12 through 14, a method for determining corresponding elements, in particular corresponding spatial points, in two descriptions of an object under measurement will now be explained as is used in step (2) of the method according to the present invention. The goal is the determination of pairs for which a clamping error can be determined as explained above. The determination of pairs is carried out in several steps and can start from each of the two descriptions of the object under measurement. To this end,

elements of one of the two descriptions are selected in advance as selected elements.

[0047] Sets MI and MS are allocated selected values. The manners of procedure of the method for determining corresponding elements, which will be explained in greater detail in the following, serves also for the interactive generation of the selected values for MI and MS by the user. Generally, no links exist yet between the selected values of MI and MS. A pair which can be already derived here, for example fitted-in cylinder against designed cylinder, can be established by the user. If enough such predefined pairs exist, then it is possible for the initial transformation to be determined automatically. These selected values of sets MI and MS clearly show where the fitting-in will be carried out or on the basis of which data set an error measure determination will be carried out.

[0048] During the determination of the registration transformation, the corresponding target values for the selected values are determined as far as this is possible. The target values are allocated to the in each case other set so that they extend this set. Pairs form MI and MS are built up.

[0049] MI and MS constitute a data concentration of the measurement or nominal description, respectively. This limitation to the essential part finally results in the advantages according to the present invention:

- reliable handling of this registration method,
- fast determination of the transformation or of the clamping error;
- generation of the selected values on the basis of the, in each case, more suitable data set, nominal or actual, combined with the automatic breaking up of the correspondence,
- noise reduction via the fitting-in of geometric elements,
- noise reduction by modeling.

[0050] The basis for the determination of pairs are:

- the indication of selected elements,
- the indication of control parameters which permit the determination of an individual subset of the target set for each selected element,
- information on the object description (data set)
    - general measured values.
    - feature elements, circle, cylinder ...
    - CAD description
    - etc..

[0051] The target set for MI is composed of:

- all design elements of the CAD description,
- arbitrary features extracted from the CAD description,
- elements which are newly designed or altered on the basis of the CAD description, in particular of elements of set MS.

[0052] The target set for MS includes:

- all measured data,
- all geometric elements such as circle, etc., which are extracted from the measured data, also from the measuring aids,
- also the elements from MI.

[0053] For the determination of pairs, a subset of the target set is determined on the basis of a selected element. The subset of the target set can include, for example, one or several spatial points, a surface description, or also a feature element. By fitting in geometric objects such as a cylinder, a plane, a sphere, or a circle, which represent a model of the object under measurement in the region of the subset, it is possible to further restrict the subset for determining the target element. In this manner, it is possible to determine corresponding elements of two descriptions of the object under measurement even if the descriptions are overdetermined.

[0054] The selection function with the aid of which a subset of the target set is determined, and the scanning function which is used to determine the point and direction of the target value from the subset, can be individually adjusted for each selected value. In particular, as already mentioned, the automatic, controlled fitting-in of geometric elements onto measured data or the design can be initiated by the method for determining corresponding elements.

[0055] The selection functions from the target sets permit:

- the selection of a subset by specifying the target space,
- und/or the selection of a subset by specifying the target type such as axis, point, surface, measuring points, etc.,
- und/or the selection by specifying features such as name, color of the visualization,
- the fixed allocation of a geometric element,
- the fixed allocation of a selected element, of course none from the same set,
- no allocation, that is the error zero is always determined for a non-evaluable element of the selected set.

[0056] The scanning functions from the subsets yield, inter alia:

- the point and/or direction of selected geometric elements, the geometric elements being fixed,
- the point and/or direction of automatically fitted-in geometric elements, the fitting-in being automatically repeated during the determination of the transformation,
- the projection point from modeling and projection of the transformed selected value as well as the projection direction from the selection input and/or model,
- the measuring point lying closest,
- etc.

[0057] The now possible fixed correlation of nominal-actual pairs permits the automatic determination of a pre-orientation T_{init}. This fixed correlation also allows axes, straight lines, normals, etc. to be paired without taking a location into account. Formerly, the indication of a

location was mandatory for the scanning. In this manner, it is possible to define pairs which exclusively describe an angle error. This enables the alignment with respect to conditions such as “is parallel to”, is perpendicular to”, etc.

[0058] This purely mathematical scanning of the measured point data and determination of the selected values of MI can be mapped onto tactile coordinate metrology without difficulty. In this context, the digitalization of measuring elements as, for example, point, sphere, cylinder, etc., corresponds to the determination of a selected value for MI. The determination of a target value for MS corresponds to the automatic point or feature measurement.

[0059] In the exemplary embodiment depicted in Figures 12 through 14, a spatial point P_v was selected as a selected value. Spatial point P_v is allocated a spatial direction N_v which, however, does not have to be always the case. Fig. 12 shows a set of points M which constitutes the target set for spatial point P_v. A target element corresponding to spatial point P_v is now to be determined from this target set M. The target element is also intended to be a spatial point which can possibly be allocated a direction in space.

[0060] Via a control parameter allocated to the selected element, a subset is determined from target set M, the intention being for the subset to satisfy the further criteria as, for example, that the subset has to lie within a specific volume or to satisfy a measure of quality. In the exemplary embodiment described here, the control parameter allocated to spatial point P_v is a specified volume, namely information on the form and position of a search cylinder Z relative to spatial point P_v. Fig. 13 shows the restriction of target set M depicted in Fig. 12 via search cylinder Z.

[0061] The target element is now determined from the subset. If the selected element possesses a direction, as in the case which is described here, then this direction can be included in the determination of the target element. To this end, here, a plane E is fitted into the region of search cylinder Z as a model of the object under measurement in an intermediate

step. Determined as the target element is the projection of the selected element, of spatial point P_v, along the allocated spatial direction N_v onto plane E, or the point of target set M which lies closest to this projection. Moreover, the target element is allocated the orientation of plane E as spatial direction.

[0062] Fig. 14 shows the projection of spatial point P_v onto plane E, a modeling of the subset defined by search cylinder Z, along spatial direction N_v. Here, the projection point constitutes the required target element, spatial point P_z, and plane orientation R is allocated to spatial point P_z as spatial direction N_z. Alternatively, the base point of the perpendicular line, namely the projection of the selected element, spatial point P_v, along plane orientation R onto plane can be determined as target element. If no spatial direction was allocated to the selected element P_v, but a spatial direction N_z was determined in connection with the target element, then the selected element P_v can be allocated N_z as the new spatial direction.

[0063] At this point, however, it should be clearly pointed out that both the target set and a suitable subset of the target set and a target element from the subset of the target set can also be determined in a different way than it is explained in the above exemplary embodiment. Within the scope of the method according to the present invention, it is possible to put together a suitable selection chain for each combination of selected element and target element.

[0064] A method for determining a transformation for aligning an object under measurement according to its nominal description will now also be explained with reference to Figures 15 through 20. Used within the scope of the determination of such a transformation are both the method for determining a clamping error described in connection with Figures 7 through 11 and the method for determining corresponding elements of two descriptions illustrated in connection with Figures 12 through 14.

[0065] In the determination of a transformation for aligning an object under measurement, the intention is to correct the position error, the tilt error as well as the scaling error.

[0066] The method is based on:

- selected elements of actual description MI of the object under measurement,
- selected elements of nominal description MS of the object under measurement,
- all selections of the above-described method for determining the clamping error,
- all selections of the above-described method for determining pairs for MI and MS,
- the specification of all components of the clamping error which are to be corrected (constraint alignment). These are, inter alia, translation, rotation, and scaling in X-, Y-, and Z-direction,
- limit values and termination criteria for the exploration of the transformation.

[0067] In a first step, an initial transformation T_{init} is determined. If parameters of the required transformation can directly be determined from the selected elements of nominal description MS and of actual description MI, it is possible to initialize selected parameters. Otherwise, the unit matrix is specified for T_{init}. In Fig. 15, it is depicted, for example, how the translatory component of the required transformation is approximated from the centroids of the selected elements of nominal description MS and of actual description MI. However, a translation and/or a rotation between corresponding feature elements of actual description MI and of nominal description MS could also be determined as the initial transformation T_{imt}.

[0068] In a second step, the selected elements of nominal description MS are oriented as the target set with respect to actual description MI via the inverse T_{imt}^{-} of initial transformation T_{init}, and the selected elements of actual description MI are oriented as the target set with respect to nominal description MS via initial transformation T_{init}. The application of these transformations to nominal description MS and actual description MI results in transforms MSt and MIt, respectively. Fig. 16 shows the orientation of the selected elements of nominal description MS and actual description MI via T_{init}.

[0069] At this point, first of all, the target elements for the selected elements of transformed nominal description MSt are determined from actual description MI and, secondly, the target

elements for the selected elements of transformed actual description MIt are determined from nominal description MS, using the method for determining pairs which is described above in detail. If no sufficient quantity of pairs can be generated, the alignment fails, that is transformation T_{init} has turned out to be unsuitable. In this case, the method for registering is to be preceded by a pre-alignment, which can be carried out, for example, interactively by the user. Mostly, an initial transformation can be definitely preset.

[0070] The found target elements are now retransferred to nominal description MS and actual description MI. The target elements of MSt are transformed with T_{init} and form the target elements for the elements of nominal description MS. Correspondingly, the target elements of MIt are transformed with T_{init}^{-} and form the target elements for the elements of actual description MI. Thus, the result reflects the situation subsequent to the application of T_{init}. Fig. 17 shows the pairs found within the scope of the determination of pairs from nominal description MS and actual description MI, making allowance for transformation T_{init}.

[0071] By determining a scalar clamping error AFt, the result of transformation T_{init} is weighted, which is shown in Fig. 18. Thus, clamping error Aft describes the quality of transformation T_{init}.

[0072] According to the present invention, initial transformation T_{init} is now modified using an exploratory method such as interval halving, gradient analysis, or Newtonian method for zero point determination, which is depicted in Fig. 19. With this modified transformation it is proceeded as with transformation T_{init}. Thus, again, the modified transformation is weighted and, possibly, further modified.

[0073] This is repeated until a criterion for terminating the exploration is fulfilled. The exploration approximates clamping error AFt asymptotically to a limit value AF0. This limit value corresponds to the best possible alignment is not known beforehand. In theory, the limit value can be 0, which corresponds to a perfect, error-free alignment. Possible criteria for termination further include:

- a preset maximum number of modifications,
- the clamping error cannot be further improved or reduced,
- the clamping error is already smaller than a selected value AFv,
- etc.

[0074] Fig. 20 shows the approximation of AFt to AF0. A possible selected value AFv is shown as well.

[0075] During the iterative search for the best fitting-in, the method which will be explained in the following turns out to be advantageous, using the Fourier analysis. Initially, parameter A to be optimized of the required transformation is varied in the vicinity of its current value in a suitable manner. As a result, one obtains a set X of parameter values. The error measures are determined for the elements of X, forming a set F(X). The range of numbers defined by X is mapped in [$-\pi$.. $+\pi$], which is achieved by linear function G. Then, the pairs of values (G(x),F(x)) x from set X are subjected to a Fourier analysis to determine phase angle w of the fundamental wave. Phase angle w defines the location at which scanned error measure F(x) becomes minimal. The inverse function of G, G^{-}, maps the found phase value into the domain of definition of the explored parameter. In an iteration step, the method is sequentially applied to all parameters to be optimized. The iteration steps are carried out until a termination criterion is fulfilled.

[0076] This method presents the advantage that the new measured value is directly generated from a small number of scans of the error function. In comparison with the method which is based on interval halving, the number of time-consuming evaluations of the error function is reduced. Moreover, the approximation of the error function with trigonometric functions maps the behavior of the error function in an optimum manner. Finally, it is an advantage that only one spectral line of the Fourier analysis has to be determined of which only the phase is relevant.

[0077] Thus, a measure for the dependency of the error function on the variation of a

parameter has been introduced with which it can be determined whether transformation parameters are sufficiently established by the selections or inputs.

[0078] Clamping error as defined herein also includes a spreading, stretching, mounting or fixing error.